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THE LEGENDARY DOMINATION NUMBER IN GRAPHS

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Abstract— A dominating set D of a graph G = (V,E) is said to be a Legendary dominating set of G , if L(G) has a dominating set of cardinality D. The Legendary dominationnumber is the minimum cardinality taken over al Legendary domiunating set of G and is denoted by $\gamma_{le}(G)$. In this paper, $\gamma_{le}(G)$ are obtained for some standard graphs.

Keywords— Domination number, Line graph, Line domination number

1 INTRODUCTION

In this paper, G=(V,E) is a finite, undirected, simple, connected graph. In general the graph has p vertices and q edges. Terms not defined here are used in the sense of Harary[1]. The line graph of G, denoted L(G), is the graph with vertex set E(G), where vertices x and y are adjacent in L(G) iff edges x and y share a common vertex in G .This was introduced by Harary and Norman[3]

In [4], E. Sampathkumar introduced the concept of global domination number as follows: A set $D \subseteq V(G)$ is said to be *global dominating set*, if D is dominating set of G and $\bar{G}$. The *global domination number* $\gamma_g(G)$ is the minimum cardinality of a global dominating set of G.

For any real number *x*, $\lfloor x \rfloor$ denotes the largest integer less than or equal to *x* and $\lceil x \rceil$ denotes the smallest integer greater than or equal to *x*.

In this paper, we introduced the Legendary domination parameter by combining the concept of domination both in graph and Line graph. The characteristics of this parameter was studided and its exact value was found for some standard graphs say complete graph,complete bipartite graph, cycle graph, path graph, crown grap, comb graph, star graph, book graph, crown graph and helm graph.

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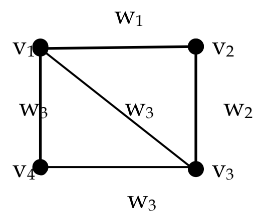
2 MAIN RESULTS

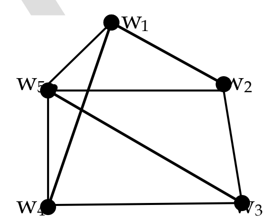
Definition 2.1

A dominating set $D \subseteq V(G)$ of graph G=(V,E) is said to be *Legendary dominating set* of G, if D is a dominating set of G and $L(G)$. The *Legendary domi nation number* is the minimum cardinality taken over all Legendary dominating sets of G and is de noted by $\gamma_{le}(G)$.

Example:2.2



G (Figure 2.1)



L(G) (Figure 2.2)

For the graph G in figure 2.1 and L(G) in figure 2.2. The vertex set D_1={ v_1 } and D_2={w_5} are the minimum dominating sets in G and L(G) respectively and hence $\gamma_{le}(G) = 1$

Theorem 2.3

For any graph G, $\gamma(G) \le \gamma_n(G) \le \gamma_{le}(G)$

Proof:

Since every Legendary dominating set of G is a neighborhood dominating set of G,

we have $\gamma_n(G) \le \gamma_{le}(G)$ ... (1)

Similarly, every neighborhood dominating set of a graph G being an vertex dominating set, we have

$\gamma(G) \leq \gamma_n(G)$ ... (2)

Then the equations (1) and (2) proves the result.

Theorem 2.4

For the complete graph K_n,

$$\gamma_{le}(K_n) = \left\lfloor \frac{n}{2} \right\rfloor = \begin{cases} \frac{n}{2} \ \ if \ \ n \ \ is \ even \\ \frac{n-1}{2} \ \ if \ \ n \ is \, odd \end{cases}$$

Proof:

Let G be the complete graph K_n , n ≥ 3

Case (i) n is even

In this case, the vertex set $S = \left\{ w_{2i} \, / \, i = 1,2,3,......,\frac{n}{2} \right\}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = \frac{n}{2}$.

Case (ii) n is odd

In this case, the vertex set $S = \left\{ w_{2i-1} \, / \, i = 1,2,3,......,\frac{n-1}{2} \right\}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = \frac{n-1}{2}$

Theorem 2.5

For the complete bipartite graph $K_{m,n}$, $\gamma_{le}(K_{m,n}) = \min(m,n)$, $m,n \geq 1$

Proof:

Let G be the complete bipartite graph $K_{m,n}$, m,n ≥ 1

Case (i) m ≤ n

In this case, the vertex set $S = \{ w_1, w_2, ...w_m \}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = m, \ m \leq n$

Case (ii) m ≥ n

In this case, the vertex set $S = \{ w_m, w_{2m}, ..., w_{m^2} \}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = n, \ n \leq m$

Theorem 2.6

For the cycle graph C_n, $\gamma_{le}(C_n) = \begin{cases} 1, & n = 3 \\ \left\lfloor \frac{n-1}{3} \right\rfloor + 1, & n > 3 \end{cases}$

Proof:

Let G be the cycle graph C_n with n ≥ 3

Case (i) n = 3

In this case, the vertex set $S = \{ w_1 \}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = 1$

Case (ii) $n \equiv 0 (\bmod 3)$

In this case, the vertex set $S = \{ w_1, w_4, w_7, ...w_{n-2} \}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = \frac{n-1}{3} + 1$

Case (iii) $n \equiv 1 (\bmod 3)$

In this case, the vertex set $S = \{ w_1, w_4, w_7, ...w_{n-1} \}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = \frac{n-2}{3} + 1$

Case (iv) $n \equiv 2 (\bmod 3)$

In this case, the vertex set $S = \{ w_1, w_4, w_7, ...w_n \}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = \frac{n-3}{3} + 1$

Theorem 2.7

For path graph P_n, $\gamma_{le}(P_n) = \left\lfloor \frac{n}{2} \right\rfloor$, $n \geq 2$

Proof:

Let G be the path graph P_n with n ≥ 2

Case (i) $n \equiv 0 (\bmod 2)$

In this case, the vertex set $S = \left\{ w_{2i-1} \, / \, i = 1,2,3,.....,\frac{n}{2} \right\}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = \frac{n}{2}$

Case (ii) $n \equiv 1 (\mathrm{mod}\, 2)$

In this case, the vertex set $S = \left\{ w_{2i} / i = 1,2,3,....., \frac{n-1}{2} \right\}$ is a minimum Legendary dominating set of G and hence $\gamma_{le}(G) = \frac{n-1}{2}$

Theorem 2.8

For the Crown graph C_n^+, $\gamma_{le}(C_n^+) = n,\ \ n \geq 3$

Proof:

Let G be a crown graph C_n^+ with at least 6 vertices. Let $\{v_1, v_2, ...v_{2n}\} \in V(G)$ from this vertex $\{v_1, v_2, ...v_n\}$ has the maximum degree in G and of same degree. Then the vertex set $S = \{v_1, v_2, ..., v_n\}$ forms a Legendary dominating set of G and hence

$\gamma_{le}(G) \leq |S| = n$ …….(1)

Let S se the γ_{le} - set of G. The dominating set in L(G) must contain n adjacent vertices in L(G) and hence $\gamma_{le}(G) = |S| \geq n$ ……(2)

Then from equations (1) and (2), we have $\gamma_{le}(G) = n$, for n≥3.

Theorem 2.9

For the comb graph P_n^+, $\gamma_{le}(P_n^+) = n,\ \ n \geq 2$

Proof:

Let G be a comb graph P_n^+ with at least 2 vertices. Let $\{v_1, v_2, ...v_{2n}\} \in V(G)$ from this vertex $\{v_1, v_n\}$ has the same degree and $\{v_2, v_3, ...v_{n-1}\}$ has the maximum degree in G and of same degree. Then the vertex set $S = \{v_1, v_2, ..., v_n\}$ forms a Legendary dominating set of G and hence

$\gamma_{le}(G) \leq |S| = n$ …………….(1)

Let S se the γ_{le} - set of G. The dominating set in L(G) must contain n adjacent vertices in L(G).

Hence $\gamma_{le}(G) = |S| \geq n$ …..(2)

Then from equations (1) and (2), we have $\gamma_{le}(G) = n$, for n≥3.

Theorem 2.10

For the star graph $K_{1,n}$, $\gamma_{le}(K_{1,n}) = 1,\ n \geq 1$

Proof:

Let G be a star graph $K_{1,n}$ with at least 2 vertices . Let $\{v, v_1, v_2, ...v_{2n}\} \in V(G)$. Let $v \in V(G)$ has the maximum degree. Then the vertex set $S = \{v\}$ forms a Legendary dominating set of G and hence

$\gamma_{le}(G) \leq |S| = 1$ …………….(1)

Let S se the γ_{le} - set of G. The dominating set in L(G) must contain only one vertex in L(G).

Hence $\gamma_{le}(G) = |S| \geq 1$ ……………..(2)

Then from equations (1) and (2), we have $\gamma_{le}(G) = 1$, for n≥1.

Theorem 2.11

For the book graph B_n, $\gamma_{le} = n+1,\ n \geq 1$

Proof:

Let G be a book graph B_n , n ≥ 1. The vertex set $S = \left\{ w_1, w_{3i} / i = 1,2,3,..\frac{n}{3} \right\}$ is a Legendary dominating set with minimum cardinality of G, since the vertex induced subgraph $\langle W / S \rangle$ is adjacent vertices of W. Hence $\gamma_{le} = n+1,\ n \geq 1$

Theorem 2.12

For the wheel graph W_n, $\gamma_{le}(W_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1,\ \ n \geq 3$

Proof:

Let G be a wheel graph W_n, n ≥ 3

Case (i) $n \equiv 0 (\mathrm{mod}\, 3)$

In this case, the vertex set $S = \left\{ w_{3i-2} / i = 1,2,3,......, \frac{n}{3} \right\}$ is a Legendary dominating set with minimum cardinality of G and hence $\gamma_{le}(G) = \frac{n}{3}$

Case(ii) $n \equiv 1(\bmod 3)$

In this case, the vertex set $S = \left\{ w_{3i-1} / i = 1,2,3,......,\frac{n-1}{3} \right\} \cup \{ w_{n+1},\ n \geq 3 \}$ is a Legendary dominating set with minimum cardinality of G and hence $\gamma_{le}(G) = \frac{n-1}{3}$

Case (iii) $n \equiv 2(\bmod 3)$

In this case, the vertex set $S = \left\{ w_{3i-1} / i = 1,2,3,......,\frac{n-2}{3} \right\} \cup \{ w_{2n,}\ n \geq 3 \}$ is a Legendary dominating set with minimum cardinality of G and hence

$\gamma_{le}(G) = \frac{n-2}{3}$

Theorem 2.13

For the Helm graph $H_n,\ \gamma_{le}(H_n) = n,\ n \geq 3$

Proof:

Let G be a helm graph H_n ,n ≥ 3. Let $\{v_1, v_2, ...v_{2n}\} \in V(G)$ from this vertex $\{v_1, v_2, ...v_n\}$ has the maximum degree in G and of same degree. Then the vertex set $S = \{v_1, v_2, ..., v_n\}$ forms a Legendary dominating set of G and hence

$$\gamma_{le}(G) \leq |S| = n \qquad \text{................(1)}$$

Let S se the γ_{le} - set of G. The dominating set in L(G) must contain n adjacent vertices in L(G).

$$\text{Hence} \quad \gamma_{le}(G) = |S| \geq n \qquad \text{.....(2)}$$

Then from equations (1) and (2), we have $\gamma_{le}(G) = n$, for n≥3.

1. CONCLUSION

In this paper, we found the exact values of Legendary domination number for complete graph, complete bipartite graph, wheel graph, star graph, book graph, crown graph, helm graph, comb graph.

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